Problem 1– Solving absolute value equations

You can use the properties of equality to solve an absolute value equation.

$$|x| + 6 = 10$$
$$|x| + 6 - 6 = 10 - 6$$
$$|x| = 4$$

$x = 4$ or $x = -4$

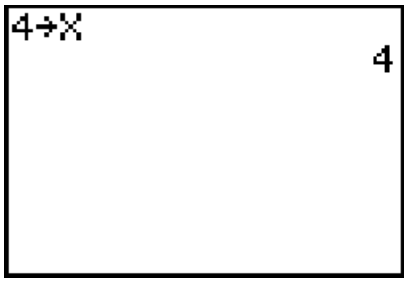
Linear absolute value equations have 0, 1, or 2 solutions.

A variable expression inside the absolute value bars can be positive or negative. To solve an absolute value equation, write it as two equations and solve them.

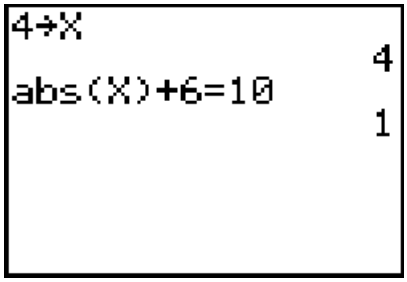
$$|x - 1| = 5$$

$x - 1 = 5$	or	$x - 1 = -5$
$x - 1 + 1 = 5 + 1$	or	$x - 1 + 1 = -5 + 1$
$x = 6$	or	$x = -4$

You cannot use the graphing calculator to directly solve absolute value equations, but you can use it to check your answers. To check the solutions $x = 4$ or $x = -4$ for $|x| + 6 = 10$, first press 4 STO▸ X,T,Θ,*n* to store 4 as x.

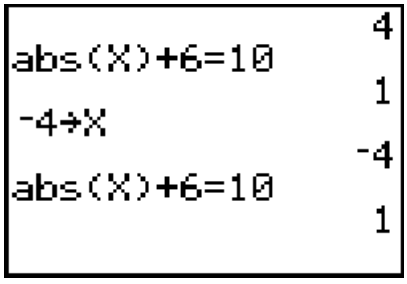


Then test the equation. The **abs(** command is found in the **MATH > NUM** menu, and the equals sign is found in the **TEST** menu. If the calculator returns a value of 1, the equation is true for the current value of x. If the calculator returns a 0, the equation is not true for the current value of x.



Check the solution $x = -4$.

- Check the solutions $x = 6$ and $x = -4$ for $|x - 1| = 5$.



Exercises

Solve each equation. If there is no solution, write no solution. Check your answers.

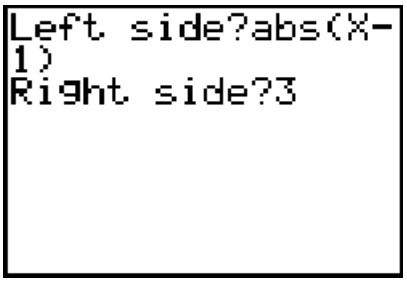
1. $|x| + 5 = 7$ **2.** $|x - 8| = -5$ **3.** $2|x| + 3 = 11$ **4.** $|x + 2| = 6$

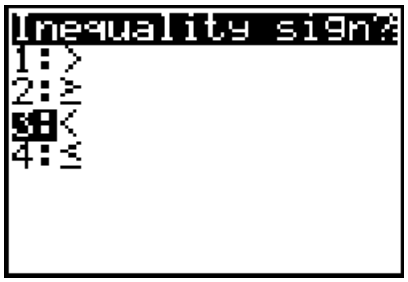
5. $|x| - 8 = -3$ **6.** $|x + 2| = 0$ **7.** $|3 - x| = 9$ **8.** $|2x - 3| = 7$

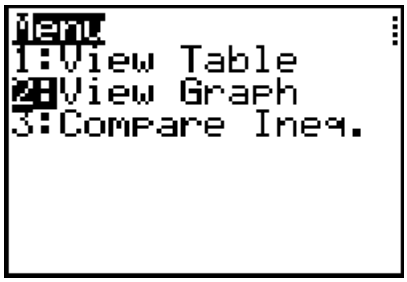
Problem 2 – Absolute value inequalities

You can write absolute value inequalities as compound inequalities. To see this, graph some absolute value inequalities in a single variable.

Run the program **ABSINEQ** and enter $|x - 1| < 3$. To do so, enter **abs(x–1)** as the left side, **3** as the right side, and choose **<** as the inequality sign. Then choose **View Graph** to graph this inequality.

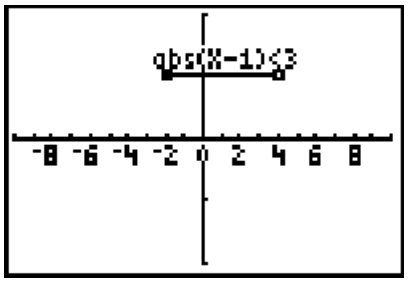




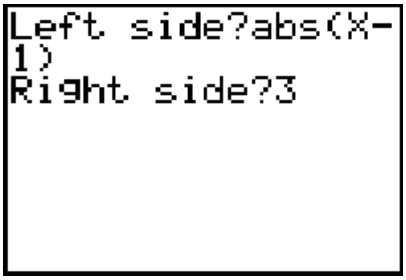


Examine the graph. $|x - 1| < 3$ means all numbers less than 3 units away from 1, so $-3 < x - 1 < 3$.

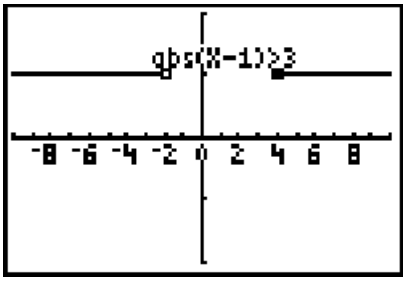
Caution: In some graphs, the open circle will appear to be filled in. This is because of the size of the pixels on the graph screen. For this reason, a "closed circle" is shown as a cross, and an "open circle" as a open or closed square.



Press CLEAR to exit the graph screen and ENTER to run the **ABSINEQ** program again. Use the calculator to graph $|x - 1| > 3$.



The inequality $|x - 1| > 3$ means all numbers more than 3 units away from 1, so $x < -2$ or $x > 4$. In general, by looking at the inequality sign, you can choose how to write an absolute value inequality as a compound inequality.



Rules for absolute value inequalities

Rule 1 An inequality of the form $|A| < b$ is equivalent to $-b < |A| < b$.

Rule 2 An inequality of the form $|A| > b$ is equivalent to $A < -b$ or $A > b$.

Rule 3 An inequality of the form $|A| \leq b$ is equivalent to $-b \leq |A| \leq b$.

Rule 4 An inequality of the form $|A| \geq b$ is equivalent to $A \leq -b$ or $A \geq b$.

Exercises

Match each absolute value inequality with an equivalent compound inequality.

1. $|x| < 3$

2. $|x| > 6$

3. $|5x| \leq 30$

4. $|x + 18| \geq 12$

5. $|x| < 6$

6. $|x| + 2 > 4$

a. $x < -6$ or $x > 6$

b. $x + 18 \leq -12$ or $x + 18 \geq 12$

c. $-3 < x < 3$

d. $x < -2$ or $x > 2$

e. $-6 < x < 6$

f. $-30 \leq 5x \leq 30$

Write each absolute value inequality as a compound inequality.

7. $|x + 7| > 9$

8. $|3x| \leq 6$

9. $|x| - 3 > 7$

10. $|2.5x| < 4$

Problem 3 – Solving absolute value inequalities

Sometimes it is necessary to simplify the absolute value inequality before writing it as a compound inequality.

$$4|x + 1| < 16$$

$$\frac{4|x+1|}{4} < \frac{16}{4}$$

$$|x + 1| < 4$$

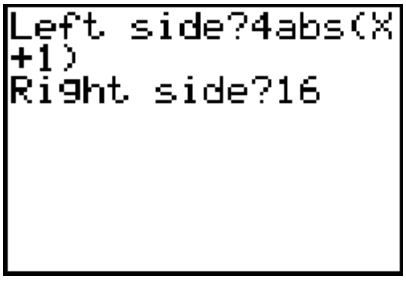
$$-4 < x + 1 < 4$$

$$-4 - 1 < x + 1 - 1 < 4 - 1$$

$$-5 < x < 3$$

You can graph absolute value inequalities to check your answers. Compare the graph of the original (unsolved) inequality with the solution.

- Use the **ABSINEQ** program to graph 4|x + 1| < 16 and compare its graph with the solution given above.



Exercises

Solve each inequality. If there is no solution, write no solution. Check your answers.

1. $|x + 8| \geq 3$ **2.** $|x - 2| \leq 1$ **3.** $|x - 3| \leq 4$ **4.** $|2x - 5| > 9$

5. $|2x - 3| \geq 7$ **6.** $|x + 2| > 0$ **7.** $|3 - x| < 9$ **8.** $-3|x + 2| > -12$