# WORD GAMES

**Teachers Teaching with Technology™**
Professional Development from Texas Instruments

TI-Nspire™    Investigation    Student    75 min

## Scrabble™

The corner for 35th Avenue and 81st Street in Jackson Heights New York bears a wonderful tribute to Alfred Mosher Butts, the inventor of the board game, Scrabble.

JACKSON HEIGHTS HISTORIC DISTRICT
35T$_1$H$_4$ A$_1$V$_4$E$_1$N$_1$U$_1$E$_1$

Alfred originally called the game "Lexiko" (Greek: "many words") but later changed it to "Criss Cross Words". Alfred contacted numerous game manufacturers, all of them rejected his game, he eventually sold the rights to entrepreneur James Brunot. James manufactured the game for some time in an abandoned school and re-branded the game as Scrabble. Despite selling a few thousand games in 1949, James made a loss, however, by 1952 James and his wife could no longer keep up with orders. One hundred and fifty million sets later, Scrabble is one of the most popular board games in the world!

## Gathering Data

To ensure the accessibility of the letters in the game reflected those used in day to day language, Alfred examined the letter frequency from a New York Times article. Your task is to run your own comparison and analysis.

Open the TI-Nspire file:  **WordGames**

Navigate to Page 1.3 (Calculator Application)

Press the VAR key and select the Scrabble program. You are provided with the option to create a new set of data (N) or add to existing data (Y). The first time you run the program you should type "N" to create a new set of data. A textbox is then provided for you to enter a sentence. Punctuation, numbers and special characters will all be ignored, only letters will be counted.
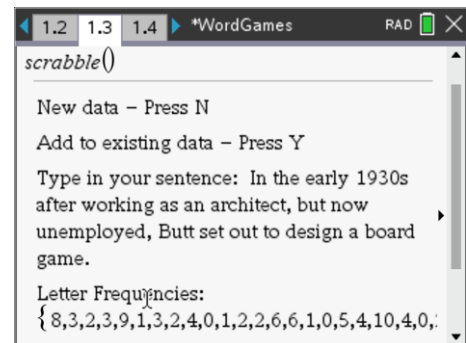
> ⚠️ A sample sentence has been included (above), an extract from the Wikipedia article on Alfred Mosher Butts. It is important to ensure your selected article does not include too many repetitions of a specific word such as a person's name as this will adversely bias the results.

After you have entered your first sentence, press ENTER to automatically run the Scrabble program again, this time enter "Y" to add to your existing data set. The original sentence will be over-written but the data is accumulated.

**Note**: You can type in shorter entries to make typing easier and simply select "Add to existing data" each time.

Author: P. Fox

TEXAS INSTRUMENTS
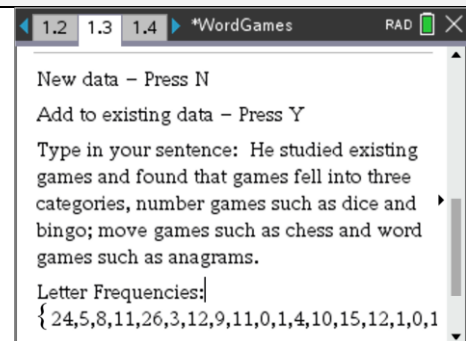
After you have entered several sentences, you can check the total character count via the Calculator application.

Type in:  sum(

Select the "freq" variable from the Variable drop down list: vars

The result represents the total number of letters counted, lower and upper case. The example opposite shows that 217 letters have been counted from the text entered to date.

| 1.2 | 1.3 | 1.4 | ▶ | *WordGames | RAD ▯ ✕ |

categories, number games such as dice and
bingo; move games such as chess and word
games such as anagrams.

Letter Frequencies:
{24,5,8,11,26,3,12,9,11,0,1,4,10,15,12,1,0,1

Done

sum(freq)                                    217

Letter frequencies are automatically generated by the program and stored in the 'freq' list in the spreadsheet application on page 1.2. The lists also include the standard score for each letter in the game of Scrabble (score) and letter frequency in the actual game (game_freq).
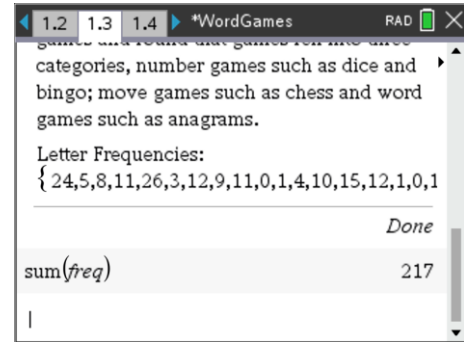
| 1.2 | 1.3 | 1.4 | ▶ | *WordGames | RAD ▯ ✕ |

| | A letters | B freq | C score | D game... |
|---|---|---|---|---|
| = | | | | |
| 1 | a | 30 | 1 | 9 |
| 2 | b | 7 | 3 | 2 |
| 3 | c | 13 | 3 | 2 |
| 4 | d | 17 | 2 | 4 |
| 5 | e | 43 | 1 | 12 |

A1  "a"

## Question: 1.

Record the letter frequencies from a sample of more than 1000 letters.

**Answer**:  Letter frequencies will vary from student to student depending on the sentences they used, they will generally resemble the letter frequencies in the game of Scrabble with the exception of the letters U and S. Butts increased the frequency of the letter U to provide more opportunities to use the letter Q and reduced the frequency of the letter S as the forming of plurals was considered to easy to generate a higher point score.

Teachers may request students record their sentences to help spot any anomalies in results. To make the lesson even more memorable, collect inspirational quotes and have students enter their favourites. As per advice to students, avoid having the same word appearing to often. (Example: Mathematics or Education)

To check student data entry accuracy, students may be given a common piece of text for their first sample:

"The purpose of education is to replace an empty mind with an open one."

Samples:
"An investment in knowledge pays the best interest"
"The more you make it easy for your kids, the more you remove the struggle necessary for learning"
"A mind when stretched by a new idea never returns to its original dimensions"
"It is impossible for a person to learn what they think they already know."

## Question: 2.

Express each of the letter frequencies as a percentage.

**Answer**: Students may use the Spreadsheet application to determine the frequencies or graph them as a histogram to see them calculated automatically. Results will vary but will be based on results from Question 1.
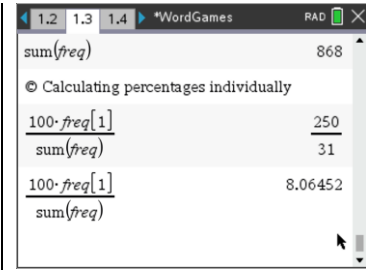
Author: P. Fox

TEXAS INSTRUMENTS

Some options for calculating percentages:
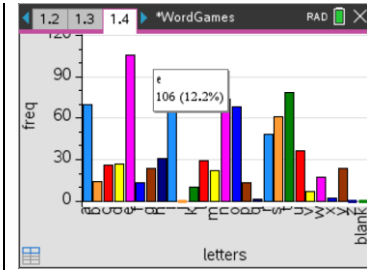
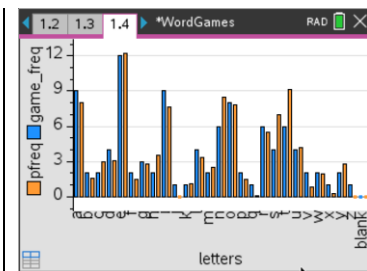| Data & Statistics Application | Spreadsheet Application | Calculator Application |
|---|---|---|
| More than 800 letters in the sample, hover over each column to see raw data and percentage. | Use a column formula. But using 'sum(freq)' in the formula, adding new letter data will automatically update the percentages. | Use either the raw values or reference them using the variable. Sample for the letter A shown above. |

## Question: 3.

The letter frequencies in the game of Scrabble have been included below. Compare your results with those from the board game, comment on any *significant* differences.

| A₁ | B₃ | C₃ | D₂ | E₁ | F₄ | G₂ | H₄ | I₁ |
|---|---|---|---|---|---|---|---|---|
| 9 | 2 | 2 | 4 | 12 | 2 | 3 | 2 | 9 |

| J₈ | K₅ | L₁ | M₃ | N₁ | O₁ | P₃ | Q₁₀ | R₁ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 4 | 2 | 6 | 8 | 2 | 1 | 6 |

| S₁ | T₁ | U₁ | V₄ | W₄ | X₈ | Y₄ | Z₁₀ | ☐ |
|---|---|---|---|---|---|---|---|---|
| 4 | 6 | 4 | 2 | 2 | 1 | 2 | 1 | 2 |

**Data & Statistics Application**

Using percentage calculations in the spreadsheet we can see the orange (data) and blue (scrabble) columns are mostly very similar. There are some stand out differences in this data set, specifically N, S and T. The S is not surprising for reasons mentioned previously, the N and T however are unusual, but support the reason why it is important for students to include their sentences so they can be checked for possible bias.

**Spreadsheet Application**

Students can do a visual comparison checking the "game_freq" and "pfreq" variables or create a difference column (as above) and use this to identify the average difference (would need to square or do absolute difference, interesting discussion) or the biggest differences.

For most letters the differences are really quite small.

Author: P. Fox

TEXAS INSTRUMENTS

**Question: 4.**

Compare letter frequency to letter score, comment on your findings.

**Answer:** There are numerous ways students can answer this question, the general trend however is nicely summarised in a graph. The more frequently the letter appears in day to day language, the easier it is to use the letter. Less frequently occurring letters are harder to get rid of and therefore attract a higher score.

The idea of this question is to see if students naturally make use of the graphing options on their calculator to present their arguments using the mathematical tool(s) they have available. This is in contrast to specifying the use of a graph and therefore simply pressing buttons as a means to an end.
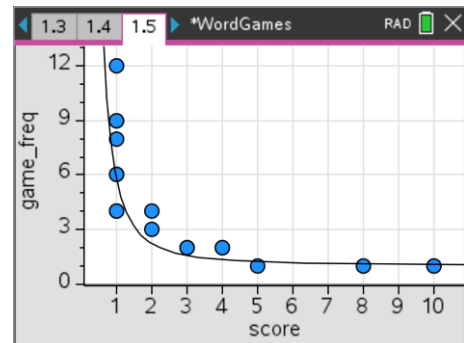
**Question: 5.**

The letter tray above includes the seven most common letters in scrabble.  If it is your turn first, what is the longest word that you can make using these letters?

**Answer**: There are many words students may put forward, including: ration, retain, atoner, orient, retina and ornate. It is important to note that as the first word, there are no other words to build on; if there were, an "D", "H", "L", "C" … would allow the player to dispose of all their letters in one turn: rationed, antihero, relation, reaction … and many more such as: ordinate.

**Teacher Note:** A discussion about how many arrangements are possible is valuable here. With 7 letters there are 7! ways of arranging the letters, but we don't have a single word that uses all of the letters without introducing additional letters.
Students may also note that letters often appear in combinations such as "tion", as in rationed, relation, reaction. This sort of observation will be helpful for the second part of this activity.

Navigate to the Lists and Spreadsheet application.  Navigate to the top of the spreadsheet and then continue pressing the UP arrow key until the first column is highlighted.

Hold down the shift key and arrow across to highlight all the columns.

Press menu and select **Actions** > **Sort**

Author: P. Fox

TEXAS INSTRUMENTS
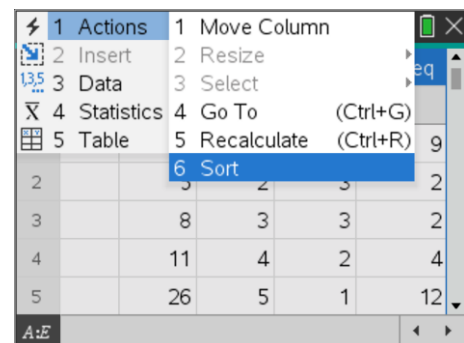
The frequency is the data that is to be prioritised, so this is the column to "Sort By".

[In this document column B holds the frequency data.]

Sort the frequencies in descending order (highest to lowest). All other columns will adjust accordingly, maintaining row correspondence.
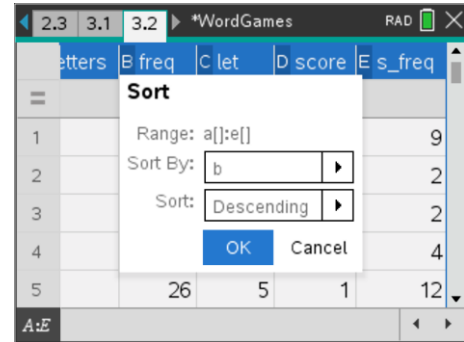
## Question: 6.

Repeat question 5 using the seven most common letters according to your data.

**Note**: In the unlikely event that your top seven letters are exactly the same as Scrabble, swap out one of the top seven with your eighth most common letter.

**Answer:** Answers will vary, in the sample(s) generated here the top 7 letters were: e, t, n, a, o, i and s which is almost identical to the top 7 for Scrabble (S swapped for R).

The highest number of letters for this combination is 6:  atones, onsite or seitan.

# TI-rdle

The TV show Lingo and more recently the online game Wordle, and all its variants, combine the popular board game Mastermind™ from the 1970's with the English language. Mastermind is a two-player game. One player creates a code using a hidden combination of coloured pegs, the other player is the code breaker. With each attempt at breaking the code, the code-breaker receives information about which pegs are the right colour and right location and which pegs are simply the right colour (wrong location). Wordle replaces colour pegs with letters, but the letters must form a word, consisting of 5 letters. Wordle players therefore have two advantages over Mastermind players:

a)    Some letters are more likely to appear in the code than others.

b)    Letters must form words.

Prior to attempting the following questions, try a couple of games of TI-rdle.

Open the TI-Nspire file:  **staTIgram**

Navigate to Page 1.2 (Python Shell)

Press Ctrl + R to run the game.

The first word attempted in this puzzle was "STORE". Three letters (S, T and O) are greyed out, this means they are not in the solution. The R is highlighted in green, this means it is the right letter in the right location. The E is highlighted in yellow, this means it is in the solution, but is currently in the wrong location.

The second word: "HEARD" has three letters that appear in the solution. The letters: E and R are now in the correct location. The letter A is in the solution, but currently in the wrong location.

The keyboard at the bottom of the page shows all the letters that have been used to date. The colour coding is also significant here, at a glance you can see which letters use and which ones have not.

Author: P. Fox

TEXAS INSTRUMENTS
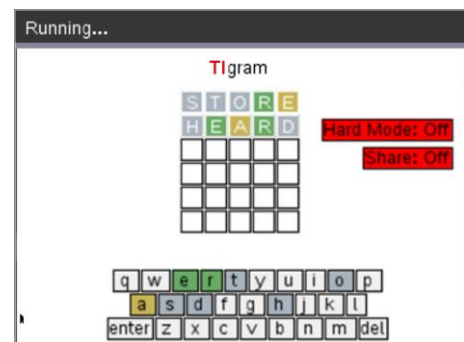
**Question: 7.**

The letter A can go in two other locations, at the start or end of the word. The following words start with AE:

| AESOP | AECIA | AEGIS | AEGIR | AEDES | AERIE | AEON |
|---|---|---|---|---|---|---|
| A person who tells or invents fables. | Small cup like structure on a specific fungi type. | Protection or support of a particular person or organisation | A high wave caused by tidal flow | Particular genus of mosquito | A nest high on a cliff top | An indefinite period of time |

Aside from the somewhat obscurity of most of the words above, which (if any) could be a solution?

**Answer**: None of these words present a possible solution as none of them have the R in the correct location.

**Question: 8.**

Based on your response to the previous question, how many possible locations are there for the letter A in this puzzle?

**Answer**: The only location for the A is now at the end of the word, this means the word appears as _E_RA.

**Question: 9.**

There are more than 20 possible five letter words that end in RA, but only three of them have an E as the second letter: TERRA, TETRA and ZEBRA. Which one is the correct answer? (Explain)

**Answer**: The first word showed that a T does not appear in the answer, the answer is therefore Zebra.

**Question: 10.**

Based on your letter frequency data, identify some of the best starting words.

**Answer**: The highest frequency letters from the Scrabble investigation are: E, A, T, O, N leaving the best option as "ATONE". Depending on the strategy, it may be better to also consider the next 5 most frequently used letters: R, U, S, H, I. The problem is there are no five letter words with these letters. It may therefore be useful to consider using the top ten letters in the first two words. Where this is not possible students may also consider selecting from the top 11 or 12 letters and justify their selection due to relatively minor differences in the frequency counts.

Students may also comment that their second word will depend on whether or not they identified any letters in the final solution and therefore whether or not they would use them in the second word. (See Question 11)

**Question: 11.**

In the game shown opposite the word "STAGE" was used first with only one letter correct. (E) The player chose "MOUND" as their second guess. (Hard Mode Off)



a)  MOUND is clearly not the correct answer, suggest reasons why a player might elect to choose such a word.

**Answer**: In this example the first guess only revealed one letter match, ending in E. This leaves an enormous quantity of viable words. Using a word with 5 different letters means the player will have checked a total of 10 letters in the first two words and not be constrained by having to end their second guess with the letter E.

b)  Focusing on the most frequently used letters (according to your data) and the combination of remaining letters, find words that have O as the second letter and E as the last letter and discuss might be the next best guess.

**Answer**: FORCE, VOICE, COBLE … of these the most common word(s) would be force and voice.
Note: The actual answer in this instance was "VOICE".

Author: P. Fox

TEXAS INSTRUMENTS
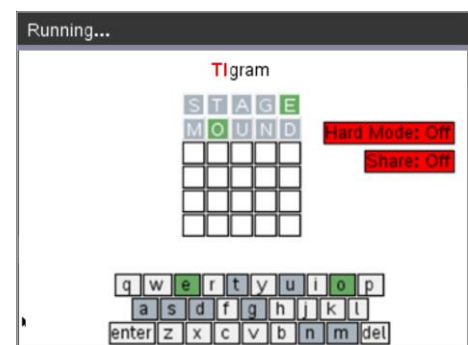
**Question: 12.**

The graph shown opposite displays a relationship between the most common letters (starting at the origin) and the quantity of words that contain the corresponding letter. For example, looking at the "letters" axis, "E" is adjacent to the origin and "Q, X and Z" are furthest away.
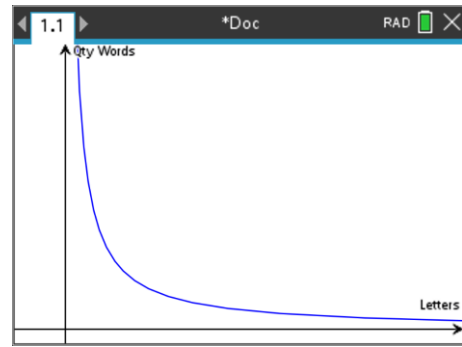


a) Emily says you are best to start with a word that only contains the most frequently used letters (close to the origin). Discuss the strengths / weakness of this strategy.

**Answer**: Emily's strategy is the most likely to get 'positive' feedback with regards to identifying specific letters that appear in the answer. The converse of this is that this strategy also leaves a lot of possible words for the answer. For example, identifying that an "E" appears in the answer leaves a LOT of words to choose from.

b) Daisy says she starts with a word that contains as many unusual letters as possible. Discuss the strengths / weaknesses of this strategy.

**Answer**: The strength of Daisy's approach is that any correct letters will leave only a handful of words to choose from, the converse is that there is a reasonably high probability that none of her letters will be contained in the solution therefore leaving a lot of words remaining.

c) Brian says he starts with a word that has four common letters and one much less frequent. Discuss this strategy.

**Answer**: Brian is taking an 'each way' bet here. If he is lucky enough to hit an unusual letter he will be well on his way to solving the puzzle, however it may also be deemed as a waste of a letter.

Author: P. Fox

TEXAS INSTRUMENTS

**Question: 13.**

Trish and Elizabeth are competing against one another on TIgram so they share their scores each day.



| Trish | Elizabeth |
|---|---|

a) Based on the data, what is the minimum number of games played by each person?

**Answer**: Trish in on a streak of 43 and her guess distribution tells us she has played 43 games, so her minimum is 43 games. Note that she may have played more, consider the situation where she couldn't solve the first 7 puzzles but has since got 43 in a row. The first 7 games do not appear in the guess distribution or the current streak.

Elizabeth has a streak of 41, but according to her guess distribution she has played 42 games. She must have played one successful game, followed by at least one unsuccessful game, then 41 successful games. This brings the total games to 43 as the minimum! She could have played more unsuccessful games prior to her winning streak of 41 games.

b) Use statistics to help determine who is the best player at this stage of the competition.

**Answer**:

Trish = E(x) = ( 2 x 2 + 3 x 15 + 4 x 16 + 5 x 9 + 6 x 1 ) ÷ 43 = 3.81

Elizabeth* = E(x) = ( 2 x 3 + 3 x 12 + 4 x 19 + 5 x 4 + 6 x 4) ÷ 42 = 3.86

*Note that this does not consider the game that Elizabeth missed. Students may include a penalty or assume that she could have solved the problem in 7 turns, however it is not known whether either player had multiple unsuccessful attempts prior to the current streak.

Elizabeth* = E(x) = ( 2 x 3 + 3 x 12 + 4 x 19 + 5 x 4 + 6 x 4 + 1 x 7) ÷ 43 = 3.93

**Question: 14.**

Russell and James have been playing the game a while and have registered the following results:



| Russell | James |
|---|---|

A quick glance at the results shows that Russell is slightly better than James. Russell describes the game scores as being a bit like golf. He describes par for any given game as '4', which would make a solution in three turns a 'birdie' and 5 turns a 'bogie'.

Author: P. Fox

TEXAS INSTRUMENTS

a) How many 'birdies' has each player scored?

**Answer**: According to the analogy, Russell has 11 birdies and James has 8.

b) What is each player's average?

**Answer**: Russell has an average of 4.29 and James has an average of 4.46

c) In golf, a handicap is determined by the difference between the players average and par. Determine the handicap for each player.

**Answer**: Russell's handicap would be 0.29 and James would be 0.46

d) Russell and James decide to apply their respective handicaps a week of games. Russell scores: 3, 5, 4, 6, 4, 4 and 3 turns. James scores 4, 4, 5, 6, 3, 5 and 5 turns. After applying their handicaps, which player is the winner for the week? [Assume the handicap is NOT re-calculated after each game]

**Answer**: Russell's score for the week: 31 ÷ 7 – 0.29 = 4.14. James's score: 32 ÷ 7 – 0.46 = 4.11. James wins by the narrowest of margins. Note that another way of calculating who wins is to determine the difference between their handicaps: 0.46 – 0.29 = 0.17. Since 1 ÷ 0.17 = 5.88, that means that approximately every 6 days James can drop a turn.

**Teacher Notes:**

There are sections (questions) in this investigation that are purposely left open. The idea is for students to demonstrate how much they are able to use mathematics to support their response. Similarly, with questions 13 and 14, students are not told how to calculate the average, therefore some students may literally write down all the scores as: 2 + 2 + 3 + 3 + 3 + 3 … and calculate the average, others may realise the quicker approach using frequency. Another option is that students may use their calculator and discover that it too has the option to reference frequency.

There are several questions where students are not specifically instructed to use graphs, however they represent a natural visual tool. Assessment of students for this task may include comments such as "naturally incorporates mathematical tools in their responses to questions". Depending on student use of technology, comments may also include: "uses technology appropriately to respond to questions without explicit instructions or references".

Some students may require assistance producing a histogram for the letter frequency. Students can use the attributes menu (Ctrl + Menu) on the y axis to add Y summary list.