Math Objectives

- Students will recognize the function $g(x) = \log_b x$ as the inverse of $f(x) = b^x$ where $b > 0$ and $b \neq 1$.
- Students will apply this inverse relationship and solve simple logarithmic equations.

Vocabulary

- exponential function
- logarithmic function
- domain
- range
- one-to-one function
- inverse function

About the Lesson

- This lesson involves the one-to-one function $f(x) = b^x$. In acknowledging the existence of its inverse, students will:
- Use the domain and range of $f(x)$ to determine the domain and range of $f^{-1}(x)$.
- Interpret the graph of $f^{-1}(x)$ as the reflection of $f(x)$ across the line $y = x$.
- Use this inverse relationship to write an equation for the graph of the inverse.
- Recognize the logarithmic notation needed to define the inverse function.
- Use the inputs and outputs of two inverse functions to complete a table and find patterns within the table.
- As a result, students will:
- Solve simple logarithmic equations and verify solutions using the corresponding exponential equations.

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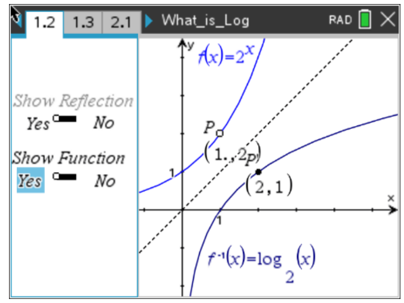
- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

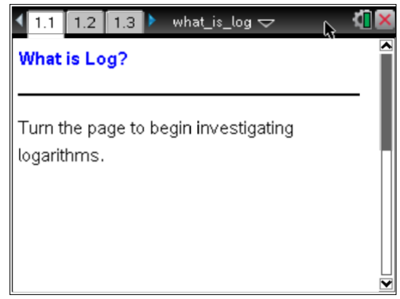
What_is_Log_Nspire_Student.pdf

What_is_Log_Nspire_Student.doc

What_is_Log.tns

Open the TI-Nspire document *What_is_Log.tns.*

You may noticed that above [10^x] is [log]. What does *log* mean? Why is [log] placed above an exponential key? You will investigate these questions in this activity.



Tech Tip: If students experience difficulty dragging a point, make sure they have not selected more than one point. Press **esc** to release points. Check to make sure that they have moved the cursor (arrow) until it becomes a **hand** getting ready to grab the point. Also, be sure that the word point appears. Then select **ctrl, center of the Touchpad** to grab the point and close the hand. When finished moving the point, select **esc** to release the point.

Tech Tip: The pages that have drag-able values have been designed to easily allow students to move the point. Instruct students to move the cursor to the open point until they get the open hand and select the **Touchpad** or press **enter**. The point should slowly blink. Then the point can be moved by pressing the directional arrows of the **Touchpad**.

Tech Tip: To move the point on a slider, tap on the point to highlight it. Then begin sliding it.

TI-Nspire Navigator Opportunity: *Quick Polls*

See Note 1 at the end of this lesson.

TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 2 at the end of this lesson.

Move to page 1.2.

1. The graph of the function $f(x) = 2^x$ is shown.
   a. What are the domain and range of $f(x)$?

Answer: The domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.

b. Recall that $f(x) = 2^x$ is a one-to-one function, so it has an inverse reflected over the line $y = x$. What are the domain and range of $f^{-1}(x)$?

Answer: The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

Teacher Tip: This may be a good time to discuss words like **invertible**. A function is invertible if each output of f is mapped from a unique input value. A function is invertible if it is a one-to-one function.

c. Point P is a point on $f(x)$. Move the Show Reflection slider to *Yes* to and then move point P. As you do so, point P' invisibly traces the graph of $f^{-1}(x)$. Since $f(x)$ can be written as $y = 2^x$, write a corresponding equation for the inverse.

Answer: $x = 2^y$

Teacher Tip: Point P and P' may not necessarily show the same number of digits, but will round to be the same.

d. The equation $x = 2^y$ cannot be written as a function of y in terms of x without new notation. Move the Show Function slider to *Yes*. The inverse of $f(x)$ is actually $f^{-1}(x) = \log_2(x)$. In general, $\log_b x = y$ is equivalent to $b^y = x$ for $x > 0$, $b > 0$ and $b \neq 1$. Why do you think x and b must be greater than 0? Why can b not be equal to 1?

Answer: x must be greater than 0 because the range of $f(x) = b^x$ is $(0, \infty)$ and thus the domain of $f^{-1}(x) = \log_b(x)$ must be $(0, \infty)$. b must be greater than 0 because negative values for b will result in negative values for x, and x has to be greater than 0. b cannot be equal to 1 because when $b = 1$, the function is linear, not exponential.

e. Move point P *so* that its coordinates are (1, 2). The point (1, 2) on $f(x) = 2^x$ indicates that $2^1 = 2$. P' has the coordinates $(2, 1)$. The point (2, 1) on $f^{-1}(x) = \log_2(x)$ indicates that $\log_2 2 = 1$. Use this relationship between exponential expressions and logarithmic expressions to complete the following table. (Move point P as necessary.)

Teacher Tip: Students will not be able to drag point *P* to all possibilities in the table. Encourage them to use the relationships.

P	*P'*	Exponential Expression	Logarithmic Expression
(1, 2)	(2, 1)	$2^1 = 2$	$\log_2 2 = 1$
(2, 4)	**(4, 2)**	$\mathbf{2^2 = 4}$	$\mathbf{\log_2 4 = 2}$
(3, 8)	(8, 3)	$\mathbf{2^3 = 8}$	$\mathbf{\log_2 8 = 3}$
(0, 1)	**(1, 0)**	$2^0 = 1$	$\mathbf{\log_2 1 = 0}$
$\mathbf{\left(-1, \frac{1}{2}\right)}$	$\mathbf{\left(\frac{1}{2}, -1\right)}$	$2^{-1} = \frac{1}{2}$	$\mathbf{\log_2 \frac{1}{2} = -1}$
$\left(-2, \frac{1}{4}\right)$	$\mathbf{\left(\frac{1}{4}, -2\right)}$	$\mathbf{2^{-2} = \frac{1}{4}}$	$\mathbf{\log_2 \frac{1}{4} = -2}$
$\mathbf{\left(-3, \frac{1}{8}\right)}$	$\mathbf{\left(\frac{1}{8}, -3\right)}$	$\mathbf{2^{-3} = \frac{1}{8}}$	$\log_2 \frac{1}{8} = -3$

Teacher Tip: Students may need to be reminded that $2^{-n} = \frac{1}{2^n}$ and thus $\log_2 \frac{1}{2^n} = -n$.

2. You have discussed the idea of reflecting the exponential function over the line $y = x$. The result of this reflection is the logarithmic function. Now we will discuss any tabular relationships that are formed between an exponential function and a logarithmic function.

   Using the first and second columns from the table above, fill in the following tables.

x	$f(x) = 2^x$
-3	$\mathbf{1/8}$
-2	$\mathbf{1/4}$
-1	$\mathbf{1/2}$
0	**1**
1	**2**
2	**4**
3	**8**

x	$f^{-1}(x) = \log_2 x$

$1/8$	**-3**
$1/4$	**-2**
$1/2$	**-1**
1	**0**
2	**1**
4	**2**
8	**3**

(a) Briefly explain your process of filling in the tables on the previous page.

Possible Answer: From the discussion earlier, the domain of function is equal to the range of the inverse of that function and the range of the function is equal to the domain of the inverse.

(b) With a classmate, discuss and describe the patterns you see in each individual column.

Possible Answer: Each input value of the function, which are also the output values of function's inverse, increase by one of the previous value. Each subsequent output value of the function, which are also the input values of the function's inverse, double the previous value.

(c) Write down a rule for each table that you can use to classify the function as either exponential or logarithmic.

Possible Answer: If the input values increase/decrease at a constant rate (increase/decrease over equal intervals), and the output values are proportional (each successive output is the result of repeated multiplication), then the function is exponential. If the input values are proportional (each successive output is the result of repeated multiplication), and the output values increase/decrease at a constant rate (increase/decrease over equal intervals), then the function is logarithmic.

Move to page 1.3.

3. Solve the logarithmic equation $\log_2 32 = y$ using the patterns from question 1. Then, use the slider to change the *n*-value to solve the logarithmic equation. How does the exponential equation verify your result?

Answer: $n = 5$ since $2^5 = 32$

Move to page 2.1.

4. Solve the equation $\log_4 \frac{1}{256} = y$. Then, use the slider to change the *n*-value to solve the logarithmic equation. How does the exponential equation verify your result?

   Answer: $n = -4$ since $4^{-4} = \frac{1}{256}$

5. Maya solved the logarithmic equation $\log_4 16 = y$. She says the answer is 4 since $4 \times 4 = 16$. Is her answer correct? Why or why not?

   Answer: Maya is not correct. The logarithmic equation $\log_4 16 = y$ is equivalent to the exponential equation $4^y = 16$. Although $4 \cdot 4 = 16$, the solution to the equation is an exponent and $4^4 \neq 16$. The correct solution is $y = 2$. Therefore, $\log_4 16 = 2$.

TI-Nspire Navigator Opportunity: *Quick Poll* See Note 3 at the end of this lesson.

6. Alex says that when solving a logarithmic equation in the form $\log_b a = y$, he can rewrite it as $b^a = y$. Is this a good strategy? Why or why not?

   Answer: Alex is not correct. There is an inverse relationship between logarithms and exponentials, but the correct exponential equation is $b^y = a$.

TI-Nspire Navigator Opportunity: *Quick Poll* See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- For all positive real *b*, where $b \neq 1$, $\log_b x = y$ if and only if $b^y = x$.

Assessment

Determine the value of the following logarithmic expressions and then justify each answer using an exponential expression.

1. $\log_3 27$
2. $\log_5 1$

3. $\log_7 7$

4. $\log_6 \frac{1}{6}$

5. $\log_4 \frac{1}{64}$

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Note 1

Question 1b and 1c, *Quick Poll:* Send an Open Response Quick Poll, asking students to submit their answer to questions 1b and 1c. If students' answers are incorrect, consider taking a Class Capture. Identify incorrect responses, briefly discussing common misconceptions. Then identify and discuss correct responses.

Note 2

Question 1c, *Live Presenter:* Consider demonstrating or have a student demonstrate how to drag and move point *P* along the graph of the function or to drag the yes/no sliders.

Note 3

Question 4, *Quick Poll:*

Send an Open Response Quick Poll, asking students to submit their answer to question 4.

Note 4

Question 5, *Quick Poll:*

Send an Open Response Quick Poll, asking students to submit their answer to question 5.