Discriminating Against the Zero

About the Lesson

In this introductory activity, students explore when a graph has two zeros, one zero, and no zeros. They will also determine when a graph has real, rational, irrational, or imaginary roots. As a result, students will:

- Use the discriminant to determine the number of zeros a quadratic function has.

Vocabulary

- discriminant
- quadratic

Teacher Preparation and Notes

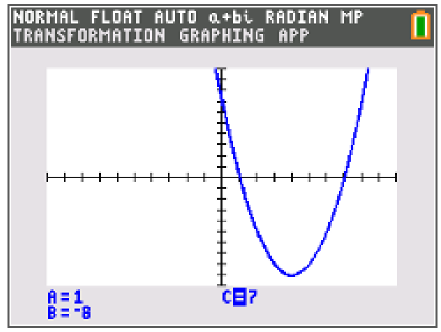
- Remind students that a solution to the equation $f(x) = 0$ is called a zero, or a root, of $f(x)$. If $x = a$ is a real solution of $f(x) = 0$, then the point $(a, 0)$ is an x-intercept of the graph $y = f(x)$.
- Before beginning the activity, make sure that students have turned off all plots and cleared all functions from the [y=] screen.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



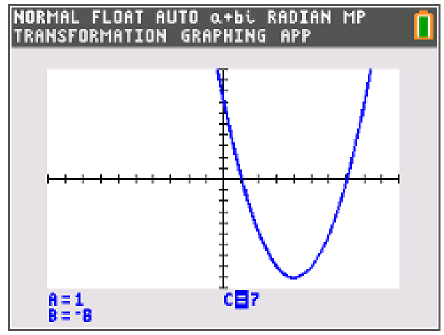
Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

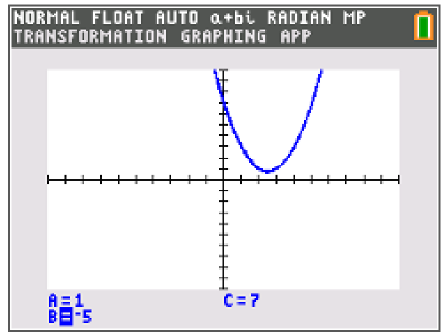
- Discriminating_Against_the_Zero_Student.doc
- Discriminating_Against_the_Zero_Student.pdf

Problem 1 – Exploring Values of *b* and *c*

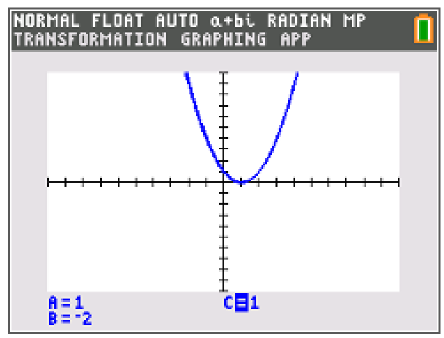


Students are to start the **Transformation Graphing** Application and enter the quadratic expression **AX^2+BX+C** into the [y=] screen for $f(x) = x^2 + bx + c$. The value of a (coefficient of x^2) should remain 1 for this part of the activity.

Have students press [window], then move up arrow to change the step size to 1, and then press [zoom] and select **ZStandard**. They can use the arrow keys to change the values or type a number and press [ENTER].



As students change the values, they are looking for instances when the graph has 2 zeros, 1 zero, and no zeros. They will also be looking to see when the zeros are rational or irrational. Specific attention should be paid to the number under the square root (called the discriminant) they are calculating on the student worksheet as they find the zeros.



If students have trouble finding functions with rational zeros or function with just one zero, give them the following values of b and c to point them in the right direction.

2 rational roots: $b = -5$, $c = 6$

1 rational double root: $b = 2$, $c = 1$

Explain to students that when a function only has one root it is called a double root. Students should also understand that there can be 1 irrational double root, but it cannot be achieved in this activity.

Students should notice that when the graph is completely above the *x*-axis, the zero command indicates "ERROR: NO SIGN CHANGE." The function has no real roots.

Ask students how they would write the functions with rational roots in factored form.

1. When does the function have two zeros? One zero? No zeros?

Answers: 2 zeros: crosses the *x*-axis twice
1 zero: touches the *x*-axis in one place
no real zeros: does not touch the *x*-axis at all

2. How does the number of zeros relate to the number under the square root?

Answers: 2 zeros: the number under the square root is positive
1 zero: the number under the square root is equal to 0
no real zeros: the number under the square root is negative

3. When does the function have zero(s) that are rational? Irrational? Not real? (Relate the type of zero to the number under the square root.)

Answers: rational: the number under square root is a perfect square
irrational: the number under square root is positive but not a perfect square
not real: the number under square root is negative

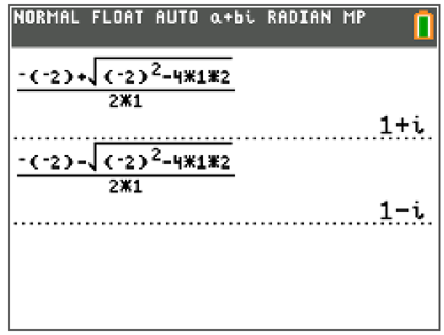
4. Give a function that has the following type of root(s). Avoid using 0 for B and C.

- 2 real, rational roots:
- 2 real, irrational roots:
- 1 real, double root (rational):
- no real roots:

Sample Answers:

2 real, rational roots: b = –8, c = 7
2 real, irrational roots: b = 3, c = –3
1 real, double root (rational): b = –2, c = 1
no real roots: b = –5, c = 7

Problem 2 – The Quadratic Formula



Students will use the quadratic formula by hand to find the exact roots of two different functions. The first function (Question 5) has irrational roots, and the second function (Question 6) has imaginary roots.

When using the quadratic formula to find the exact roots on the Home screen, the cursor will remain under the radical symbol when students are entering **$(–2)^2$–4*1*2**. To move out from under the radical symbol, students can press the arrow keys.

Tech Tip: Students should only use the fraction template with the TI-84 Plus CE, as the other graphing calculators will result in an error message.

5. Use the quadratic formula to find the exact value of the zeros of $f(x) = x^2 + 3x - 1$. What are the values of *a*, *b*, and *c*?

Answer:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot -1}}{2 \cdot 1}$$
$$= \frac{-3 \pm \sqrt{13}}{2}$$

6. By hand, use the quadratic formula to find the imaginary zeros of $f(x) = x^2 - 2x + 2$. Show your work. Remember that $\sqrt{-1} = i$.

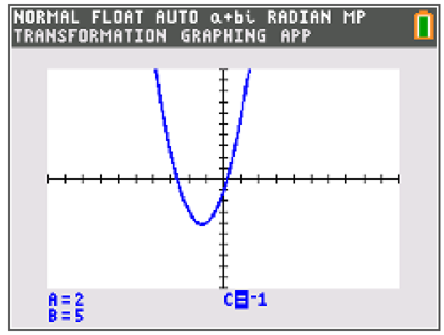
Answer:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$
$$= \frac{2 \pm \sqrt{-4}}{2}$$
$$= \frac{2 \pm \sqrt{-1 \cdot 4}}{2}$$
$$= \frac{2 \pm 2 \cdot i}{2}$$
$$= 1 \pm i$$

Problem 3 – Exploring the Value of *a*

Students will now explore the graph from Problem 1 with changes in the value of *a*. They should try both negative and positive values of *a*. You may wish to ask students why *a* should not be set equal to 0.

Students should see changes in the value of *a* do not change the conclusions they came to in Problem 1.



7. In Problem 1, **A** was set equal to 1. Do your conclusions from Problem 1 still hold if **A** ≠ 1?

<u>**Answer:**</u> Yes

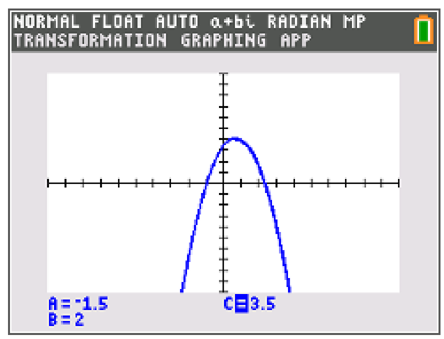
Problem 4 – Exploring Other Rational Numbers

This time students will explore the values of *a*, *b*, and *c*, but will use non-integer values as well as integers.

Remind students that repeating decimals are rational numbers.

Students may think that their conclusions from Problem 1 do not apply to this problem because numbers under the square root do not look like the typical perfect square. Have them convert the decimal to a fraction, and they will see that decimals that eliminate the square root have perfect squares in the numerator and denominator.

After students complete their investigation, a formal discussion of the discriminant should follow.



8. Do your conclusions from Problem 1 remain the same if *a*, *b*, and *c* are not integers?

<u>**Answer:**</u> Yes

9. Why do some decimals under the square root, like 12.25, make the zeros rational, but other decimals make the zeros irrational?

<u>**Answer:**</u> 12.25 in fraction form is $\frac{49}{4}$. Both the numerator and denominator are perfect squares.

Practice

Students can practice with problems from their book, using the discriminant to determine the number and type of zeros of a quadratic function.

Then, students are to use the quadratic formula to find the zeros. They can either do this by hand or with the graphing calculator.