Proportionality in Tables, Graphs, and Equations

TEACHER NOTES

MATH NSPIRED

Math Objectives

- Students will recognize and identify the characteristics of proportional and non-proportional relationships by analyzing a table, equation, and graph.

Vocabulary

- ratio
- slope
- *y*-intercept
- constant rate of change
- proportional
- non-proportional

About the Lesson

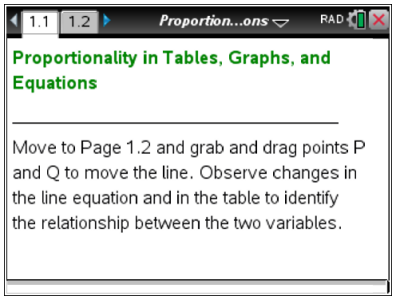
- This lesson involves analyzing changes in tables, equations, and graphs of lines to determine proportionality. Students will:
  - Explore how a change in one representation affects another representation.
  - Determine that a relationship is proportional if the ratio of the *y*-coordinate to the *x*-coordinate for each point on a line stays the same and the line goes through the origin.
  - Determine that a relationship is non-proportional if the ratio of the *y*-coordinate to the *x*-coordinate for each point on a line is *not* the same and the line does *not* go through the origin.

TI-Nspire™ Navigator™

- Class Capture to monitor students' work and generate discussion.
- Quick Poll to assess students' understanding of proportionality.
- Live Presenter to let students demonstrate and explain their work.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Proportionality_in_Tables_Graphs_and_Equations_Student.pdf

Proportionality_in_Tables_Graphs_and_Equations_Student.doc

TI-Nspire documents

Proportionality_in_Tables_Graphs_and_Equations.tns

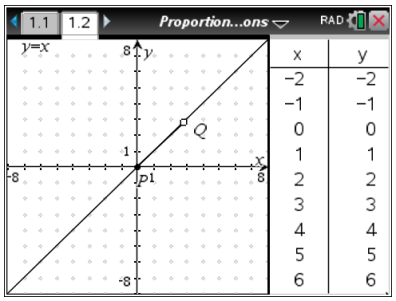
Proportionality_in_Tables_Graphs_and_Equations_Assessment.tns

Discussion Points and Possible Answers

Tech Tip: If students have difficulty dragging a point, make sure that they have moved the cursor until it becomes a hand getting ready to grab the point. Then press ctrl to grab the point and close the hand.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. Grab and drag point Q. Notice the table of values for the coordinates of the points that lie on a line.
   a. What stays the same, and what changes as the line moves?

   Sample Answers: The line always passes through the origin even though it is rotating. The numeric coefficient of x changes in the equation. (Remind students that this numeric coefficient is called the slope of the line.) All of the y-values changed except for the y-value when $x = 0$.

Teacher Tip: You might want to demonstrate how to grab point Q and drag the associated line. (Students might not realize Q is not a point listed in the table.)

2. Grab and drag point Q until you see the equation $y = 5x$.
   a. Select three points (excluding the origin) from the table, and find the ratio of the y-coordinate to the x-coordinate for each point.
   b. In the table below, record your chosen points and their ratios.

   Sample Answers: Answers will vary.

x	y	$\frac{y}{x}$

3. What do you notice about the values in the $\frac{y}{x}$ column?

Answer: The x- and y-values vary, but the value of $\frac{y}{x}$ is the same (constant) for all three points.

Teacher Tip: The slope is $m = \frac{y-0}{x-0}$ for each (x, y) point on a line through the origin.

4. Compare the values of the ratios with the equation of the line. What do you notice?

Answer: The ratio of $\frac{y}{x}$ is always equal to 5, and 5 is the coefficient of x in the equation of the line. This coefficient is called the slope of the line.

5. Why was it necessary to exclude the origin when calculating the ratio?

Answer: It is impossible to divide by zero.

6. Grab and drag point Q to plot a line with a different equation. Record the equation of this line below.

Sample Answer: Answers will vary.

7. Select three points from the table of values, and find the ratio of the y-coordinate to the x-coordinate for each point. Record your results in the table below:

Equation of the line:

Sample Answers: Answers will vary.

x	y	$\frac{y}{x}$

8. What do you notice about the values in the $\frac{y}{x}$ column?

Answer: The x- and y-values will vary, but the ratios have the same (constant) value.

9. Compare the values of ratios with the equation of the line. What do you notice?

Answer: The ratio, $\frac{y}{x}$, is the slope of the line.

TI-Nspire Navigator Opportunity: *Class Capture and Live Presenter*
See Note 2 at the end of this lesson.

10. Based on your observations and computations, what stays the same for each line?

Answer:

Table

- The quotient $\frac{y}{x}$ is the same for any point other than the origin.
- For $x = 0$ in the table, y is always equal to 0.

Equation

- The form of the equation is $y = m \cdot x$.
- The value of $\frac{y}{x}$ is the same as the value of m in the equation.

Graph

- The line passes through the origin.

Teacher Tip: If students have studied direct proportional relationships, remind them that the characteristics listed above define a direct proportional relationship between the *x*- and *y*-coordinates of the points on the line. If students have not studied proportionality, this concept should be introduced here.

Teacher Tip: Students might confuse direct proportional relationships with solving a proportion equation. Check to see if students realize they are comparing three different representations of the same information—not solving an equation.

11. Drag point P away from the origin, along the y-axis, and select a new location for it. Record the equation of this line below.

Sample Answers: Answers will vary.

12. Select three points on this new line, and find the ratio of the *y*-coordinate to the *x*-coordinate for each point.

Equation of the line:

Sample Answer: Answers will vary.

x	y	$\frac{y}{x}$

13. What do you notice about the values in the $\frac{y}{x}$ column?

Answer: The *x*- and *y*-values vary, and the value of the ratio, $\frac{y}{x}$, for each selected point is different.

14. What stays the same for each given line when P is not at the origin?

Answer:

Table

- When $x = 0$, y is always equal to b, $b \neq 0$. The values of y change by the amount P is moved (up, negative; down, negative).

Equation

- The form of the equation is $y = m \cdot x + b$ where $b \neq 0$.

Graph

- The slope (slant) of the line stays the same as P is moved.
- The ordered pair, (0, *b*), is the point where the line crosses the *y*-axis (the *y*-intercept).

15. Is *y* proportional to *x*? Justify your answer.

Answer: No. The value of $\frac{y}{x}$ is not constant.

16. Compare the $\frac{y}{x}$ ratios on the lines running through the origin and the $\frac{y}{x}$ ratios on the line that does not run through the origin.

a. In which case are the $\frac{y}{x}$ coordinates proportional to each other?

Answer: The coordinates of points on lines going through the origin are directly proportional because the ratio $\frac{y}{x}$ is constant.

b. Why?

Answer: This constant of proportionality is usually designated as *k* and is the slope of the equation which is usually designated as m in the equation $y = mx$.

Teacher Tip: Students will model with mathematics. Students will now be able to determine whether a direct proportional relationship exists between two variables from tables, graphs, or equations.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to:

- Identify linear relationships that are proportional by determining whether the graph of a line goes through the origin.
- Identify linear relationships that are proportional by determining whether the ratio of the value of *y*-coordinates to the value of the *x*-coordinates of any pair of ordered pairs on the line has a constant value.
- Identify linear relationships that are proportional by determining whether the equation of the line is of the form $y = m \cdot x$.

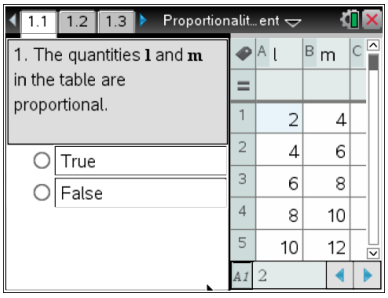
Assessment

Have students identify whether various tables, graphs, and equations represent proportional relationships between quantities. Use the **Proportionality_in_Tables_Graphs_and_Equations_Assessment.tns** document**.**

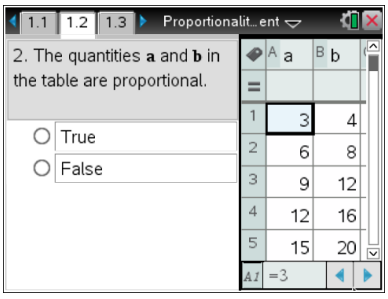
TI-Navigator Opportunity: Send and Collect Documents; Review Workspace
See Note 4 at the end of this lesson.

Answers:

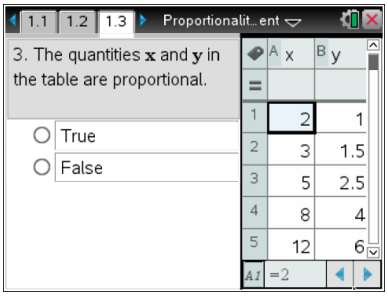
1. False. 2/4 ≠ 4/6



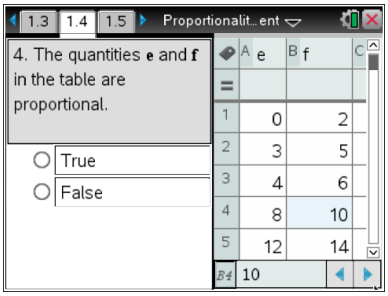
2. True. There is a constant ratio of ¾.



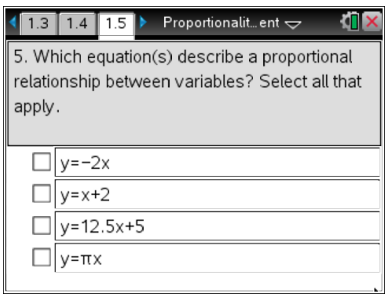
3. True. There is a constant ratio of 2:1.



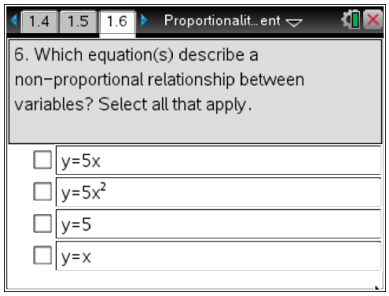
4. False. 3/5 ≠ 4/6



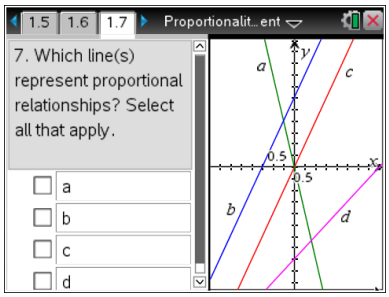
5. $y=-2x$ and $y=\pi x$



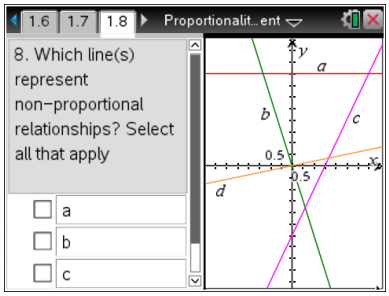
6. $y = 5x^2$ and $y = 5$



7. Lines a and c show a proportional relationship because the lines go through the origin.



8. Lines a and c do not show a proportional relationship because the lines do not go through the origin.



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Note 1

Pre-Activity, Quick Poll

Use Quick Poll for a pre-assessment before beginning the lesson. Ask students in an Open Response Quick Poll to give their definition for what "*y* is proportional to *x*" means to them.

Note 2

Question 9, Class Capture

Use Class Capture to show students' screens. In this part of the activity, students will have different lines plotted on the graphs. Group screens with like equations together. Use Live Presenter, and ask different students to explain their observations.

Note 3

Question 16, Quick Poll

A Yes or No Quick Poll can be given to ask the following questions:

- Does a line through the origin show a proportional relationship between two variables?
- Does a line that does not go through the origin show a proportional relationship between two variables?

An Open Response Quick Poll can be used as a post-assessment. Ask students to write their definition now for what "*y* is proportional to *x*" means to them and indicate whether or not their definition changed.

Note 4

Assessment, Review Workspace

Send and Collect the files from the assessment TI-Nspire document to students. As the results are shown in the Review Workspace, the entire class will see how each question was answered. A discussion about why students chose a particular answer will allow students to clarify their thinking.