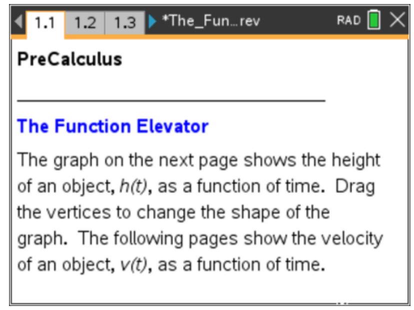

Math Objectives

- Students will create a piecewise linear graph to model a real life scenario.
- Students will identify connections between position and velocity functions and their graphs.
- Students will create velocity graphs based on information about position and position graphs based on information about velocity.
- CCSS Mathematical Practice: Students will model with mathematics.
- CCSS Mathematical Practice: Students will reason abstractly and quantitatively.

Vocabulary

- velocity
- position
- vertex

About the Lesson

- This lesson involves creating and comparing graphical representations of position and velocity functions from a scenario.
- As a result, students will:
- Manipulate pieces of a piecewise linear function to represent a real life scenario about the position of an object.
- Manipulate pieces of a piecewise linear function to represent the position of an object given information about its velocity.
- Predict and evaluate predictions about the velocity of an object based on a graph of its position.

TI-Nspire™ Navigator™ System

- Send out the *The_Function_Elevator.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- The_Function_Elevator_Student.pdf
- The_Function_Elevator_Student.doc

TI-Nspire document

- The_Function_Elevator.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, make sure they have not selected more than one point. Press esc to release points. Check to make sure that they have moved the cursor (arrow) until it becomes a hand ( ) getting ready to grab the point. Also, be sure that the word point appears. Then select ctrl to grab the point and close the hand ( ). When finished moving the point, select esc to release the point.
>
> **Tech Tip:** If you are having difficulty moving the correct point, tap once on the point to display the "select object" menu, and then select *point to move*.

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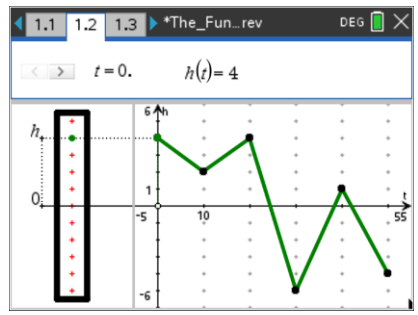


1. The graph on the right side of this page shows the movement of an elevator over time. The bar on the left side of the page shows the vertical motion of the elevator, that is, the floor the elevator is on at a given point in time.

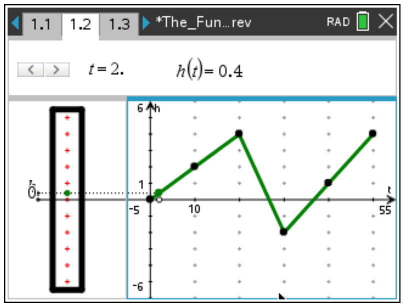
   a. Notice that the scale for $h(t)$, the height of the elevator at time t, ranges from –6 and 6. Describe a real life scenario where it would make sense for the height of the elevator to be a negative number.

   Sample answers: There are two possible scenarios that make this possible. The elevator could be in a building that has floors above ground and floors below ground, so 0 could represent ground level, while 6 represents 6 floors above ground level and –6 represents 6 floors below ground level. Alternatively, 0 could represent the floor on which one was beginning, so –6 would be 6 levels below that floor, while 6 would be 6 floors above that floor.

   b. Notice that $h(t)$, the height of the elevator at time t, is a continuous function, so it is possible that $h(t)$ is not a whole number; for example, $h(t)$ could be 1.7. Describe a real life scenario where it would make sense for the height of the elevator not to be a whole number.

   Sample answers: As an elevator descends or ascends between floors, it travels through all the space between two floors. So a height of 1.7 could represent an elevator that is partway through its ascent from floor 1 to floor 2, or partway through its descent from floor 2 to floor 1. However, an elevator would probably not stop or change direction at a height of 1.7, unless, of course, it was not operating properly.

Teacher Tip: The teacher should ensure that students understand the scale and the range of the vertical axis before continuing with the problem. It is important for students to have an understanding of negative values of $h(t)$ and of the continuous nature of the height of an elevator before proceeding with the activity.



2. Drag the vertices to transform the graph so it models the following scenario. Record a sketch of your graph, and explain how it reflects the scenario.

   Gus boarded an elevator on the ground floor. It took 20 seconds for the elevator to rise 4 floors. When Gus reached 4 floors above the ground floor, he remembered that he left his cell phone in the gym, in the basement of the building. Gus rode the elevator to 2 floors below the ground floor in 10 seconds, and was about to get off the elevator to check for his cell phone in the gym, when he reached into his pocket and found that he had his cell phone all along. He promptly rode the elevator back to 4 floors above the ground floor, taking 20 more seconds, where he got out.

   Sample answers: The elevator starts at height 0 at time 0 because Gus is starting at the ground floor. The next point on the graph is at (20, 4) because 20 seconds later, the elevator is 4 floors above the ground floor. A straight segment connects the two points because we are assuming that the elevator is traveling at a constant velocity. The next point on the graph is (30, –2) because 10 seconds later (a total of 30 seconds after beginning), the elevator is on the 2^{nd} floor below ground level, represented by height –2. Again, we connect with a straight segment assuming that the elevator travels at constant velocity. The next point is (50, 4) because it takes an additional 20 seconds to ascend to 4 floors above ground.

Teacher Tip: Teachers might want to take special care to help students interpret the graph. While the graph has segments that slant forward, this does not mean that the elevator is moving forward. The elevator moves up and down, and the change in its height is modeled over time. It is important to ensure that students understand the motion of the elevator and connect the motion to the model.

Teacher Tip: CCSS Mathematical Practice: Students will model with mathematics. Students use a graph to model a real life scenario, adjusting an existing model to fit the parameters of the scenario.

3. Is your graph the only possible graph to represent the scenario described in question 2? If so, why? If not, why not?

   Sample Answers: No. Vertical shifts of the graph would be possible. For example, one could have considered $h = 0$ to represent the initial height, the 2^{nd} floor, and all other heights of the elevator would be based on this. Furthermore, the function is piecewise linear, reflecting an assumption that the velocities are constant. An elevator would likely move with non-constant velocity (i.e. starting slow, speeding up, and slowing down before coming to a stop) so the segments connecting the elevator stops need not be straight.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

4. At what speed was the elevator moving for the first 20 seconds? For the next 10 seconds? For the final 20 seconds? How do you know?

   Sample Answers: The elevator is moving at a rate of 0.2 floors/second for the first 20 seconds, 0.6 floors/sec for the next 10 seconds, and 0.3 floors/sec for the final 20 seconds. In the first 20 seconds, the elevator rises 4 floors, so 4 floors/20 seconds, or 0.2 floors/sec. In the next 10 seconds, the elevator falls 6 floors (from 4 floors above ground to 2 floors below ground), so 6 floors/10 seconds, or 0.6 floors/sec. In the final 20 seconds, the elevator rises 6 floors (from 2 floors below ground to 4 floors above ground), so 6 floors/20 seconds, or 0.3 floors/sec.

   Teacher Tip: Students might struggle to determine units for the elevator's speed. Teachers might want to remind students that a change in floors can still be thought of as a distance traveled. It might be helpful to relate to known measures of speed, such as miles per hour.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

5. a. Luis says Gus was moving fastest 10 seconds into his trip, and Stephanie says Gus was moving fastest 25 seconds into his trip. Who is right? Why?

   Sample Answers: Stephanie is right. 10 seconds into Gus's trip, the elevator is moving at 0.2 floors/sec; while 25 seconds into his trip, the elevator is moving at 0.6 floors/sec.

b. Amanda says Gus was moving slowest 15 seconds into his trip, and Darryl says Gus was moving slowest 35 seconds into his trip. Who is right? Why?

Sample Answers: Amanda is right. 15 seconds into Gus's trip, the elevator is moving at 0.2 floors/sec; while 35 seconds into his trip, the elevator is moving at 0.3 floors/sec.

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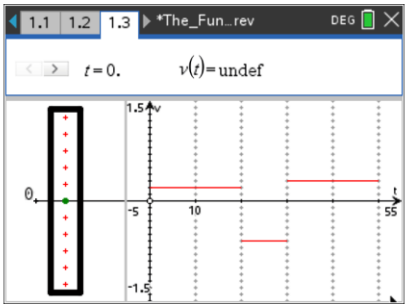

6. This graph shows the velocity of the elevator based on the graph you constructed in question 2.

   a. The velocity graph is positive in some places and negative in others. Why?

Sample Answers: The velocity graph is positive when the elevator is ascending and negative when the elevator is descending.

b. What are the connections between the scenario of Gus riding the elevator, your graph of the motion of the elevator, and the velocity?

Sample Answers: When the elevator is moving fastest, the magnitude of the velocity is greatest, and when the elevator is moving slowest, the magnitude of the velocity is least. Because the rate at which the elevator is ascending or descending is constant over the specified time intervals, the velocity is constant over corresponding time intervals.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 3 at the end of this lesson.

Teacher Tip: The discontinuity of the velocity graph indicates that velocity abruptly shifts. Of course, this is not realistic. Teachers might want to discuss this with students and ask them how a velocity graph should look to better model the movement of an elevator, particularly changes in direction. Teachers might also want to initiate a discussion of the difference between velocity and speed, particularly to help students understand negative velocities on the graph.

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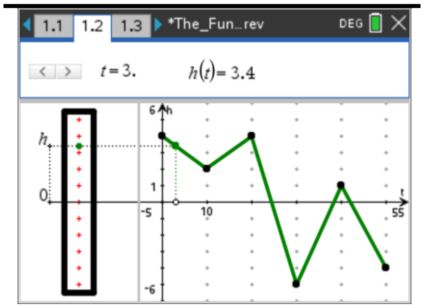
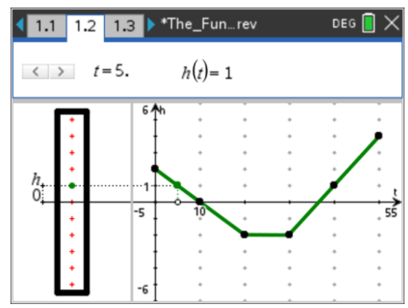


7. Drag the vertices to transform the graph so it models the height of the elevator in the following scenario. Record a sketch of your graph, and explain how it models the scenario.

   Gus and his dog left their room on the 2nd floor and took the elevator down for 20 seconds at a rate of 2 floors every 10 seconds. When the elevator doors opened, Gus's dog sprinted out of the elevator. A friendly person on the elevator held the doors for 10 seconds while Gus collected his dog. The elevator continued upward at a rate of 3 floors every 10 seconds, and Gus was so busy scolding his dog, that he didn't realize what floor he was on when he got off the elevator 20 seconds later.

   <u>Sample Answers:</u> The first point is (0, 2) because at time 0, Gus is on the 2nd floor. Descending for 20 seconds at 2 floors every 10 seconds means Gus will descend a total of 4 floors, putting him on floor "–2", or 2 floors below ground level. This is the point (20, –2) on the graph. When the elevator is held for 10 seconds, it is staying 2 floors below ground for 10 seconds and doesn't move during that time. This gives the point (30, –2) on the graph, and indicates that the segment connecting (20, –2) and (30, –2) should be horizontal, since the elevator does not move. Finally, Gus ascends 3 floors every 10 seconds for 20 seconds, making his total ascent 6 floors. This means he is 6 floors above floor –2, or 4 floors above ground. This gives the final point (50, 4).

8. What floor do Gus and his dog end up on? How do you know?

   <u>Sample Answers:</u> Gus ends up on the 4th floor. I know this because he started on the second floor, descended 4 floors putting him 2 floors below ground, and then ascended 6 floors, putting him 4 floors above ground.

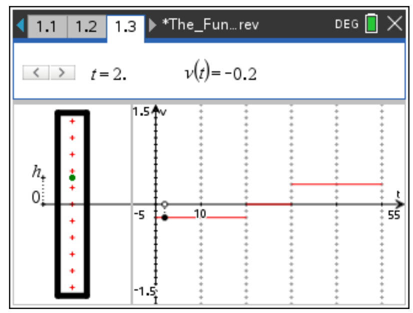
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 4 at the end of this lesson.

9. Is your graph the only possible graph to model the scenario described in question 7? If so, why? If not, why not?

 <u>Sample Answers:</u> No, it's not the only graph possible. Vertical shifts are possible, particularly if one bases the height of the elevator off of the initial height, not the floor. Furthermore, we assume that velocities are constant, but it is possible from the scenario that they might not be constant.

> **TI-Nspire Navigator Opportunity: *Class Capture***
> **See Note 5 at the end of this lesson.**

10. Draw a sketch showing your prediction of what the velocity graph will look like for the scenario modeled in question 7. Why do you think the velocity graph will look this way?



 <u>Sample Answers:</u> The velocity will be –.2 for the first 20 seconds, as the elevator is descending at a rate of 2 floors for every 10 seconds, or 0.2 floors per second. Descent indicates that the velocity is negative. The velocity is 0 between 20 seconds and 30 seconds because the elevator is stationary during that time period. Finally, the velocity is .3 between 30 seconds and 50 seconds, because for those 20 seconds, the elevator ascends at a rate of 3 floors every 10 seconds, or 0.3 floors per second.

> **Teacher Tip: CCSS Mathematical Practice:** Students will reason abstractly and quantitatively. Mathematically proficient students are able to explain how the graph of the velocity function can be determined from the graph of the position function.

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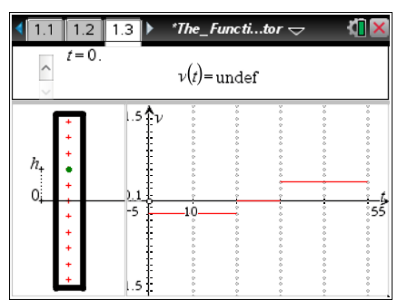

11. How does your prediction about the velocity graph compare to the actual graph? If you were incorrect, what mistakes did you make?

 <u>Sample Answers:</u> Answers will vary. Common mistakes might include neglecting the directional nature of velocity or omitting the 10 second period where the velocity is 0.

12. In general, how are the graphs of the motion of an elevator and its velocity related? Explain your thinking.

    <u>Sample Answers:</u> The graph of the velocity function is a plot of the slopes of the elevator motion function.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- That for a piecewise linear position function, the velocity function is a piecewise constant function that gives the slope of the position function.
- How to use a graph to model the position of an object from a real life scenario.

Assessment

Students write and model their own elevator scenarios, or write elevator scenarios and exchange with other students to be modeled.

Teachers create a problem that only has information about velocity and asks students to graph the position function from that information.

TI-Nspire Navigator

Note 1

Question 3, Name of Feature: Class Capture

Use a Class Capture to share students' graphs of the elevator's position. In particular, look for graphs that are different so students have opportunities to compare and contrast representations, and include different graphs which could both be correct under different assumptions.

Note 2

Question 4, Name of Feature: Quick Poll

Use a Quick Poll to determine students' understanding of the speed of the elevator.

Note 3

Question 6, Name of Feature: Class Capture

Use a Class Capture to share students' graphs of the elevator's velocity. In particular, look for graphs that are different to compare and contrast representations. If possible, teachers might want to capture students' position and velocity graphs together so that students can see that functions with differing position graphs might have the same velocity.

Note 4

Question 8, Name of Feature: Quick Poll

Use a Quick Poll to ensure that all students could determine ending position based on information about initial position and velocity.

Note 5

Question 9, Name of Feature: Class Capture

Use a Class Capture to share students' graphs of the elevator's position. In particular, look for graphs that are different to compare and contrast representations, and include different graphs that could each be correct under different assumptions.

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